TREE DIAGRAM OF POSSIBILITIES: ANALYSIS OF ITS CONSTRUCTION BY BLIND STUDENTS

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The objective of this paper was to discuss the construction of the tree diagram of possibilities by three blind students from a high school in Brazil. This activity was part of the teaching sequence "Jefferson's Random Walks," which deals with probabilistic basic concepts. In the diagram, the combinations of the visits of Jefferson to five friends (placed on a 3D 5x5 board), depending on four coin tosses, were presented. Two plastic trays were used (representing the coin), foam cards of two different textures (representing each face), and toys to represent the visited friend. Students identified just four out of the six visiting paths at the board's center. The researcher helped by identifying the missing paths, and the students registered all 16 possible paths. The importance of this study is to contribute to discussions about adaptation of materials and teaching methods for probabilistic contexts for blind students.

INTRODUCTION
        The policy of including students with special educational needs (SEN) in Brazilian schools has become more popular as a result of recommendations included in official documents and public politics focused on the construction of a school for everyone. Although inclusion is promoted by the government, there infrastructure which meets SENstudents´ needs is lacking, since access to school extrapolates the presence in classrooms and implies appropriation of the knowledge and educational opportunities offered (Brasil, 1998a).
        To resolve these difficulties, the Ministry of Education and Culture produced and distributed, among other materials, the National Curricular Parameters: Curricular Adaptations – NCP:CA (Brasil, 1998a) and the Live School Project (Brasil, 2000). These documents deal with adaptations, known as of short adaptations, that may be of help in teaching practice, allowing teachers to establish physical, environmental and material conditions for SEN students' participation in school activities. Such curricular adaptations include the adaptation of artifacts for use as didactical material for students' learning. In this context, much research remains to be done, but Ferronato (2002), Fernandes (2008), and Vita (2012) have already made significant contributions to the teaching of math to blind students.
        In particular, Vita (2012) investigated the learning of basic concepts of probability (bcP) by blind students. She claims that despite their absence in official documents there are specific curriculum guidelines for teaching this concept to blind students; she applied what was recommended in the NCPfor elementary schools (Brasil, 1998b) and curricular guidelines for high schools (BRASIL, 2006). This exercise was, in part, based on the assessment that she made of the material issued by the California Department of Education (CDE) – The Braille Mathematics Standards (CDE, 2006). According to the Brazilian documents it is the responsibility of teachers to encourage students to perform experiments, observe events, represent them with tables and tree diagrams, building sample space in situations like rolling dice, tossing coins, etc.
        Vita (2012) worked with blind students on bcP, using the tasks in the teaching sequence Random Walks of Jefferson (TS RWJ). The cbP involved were: sample space, events, probability of simple events, differences between a deterministic and a random experiment, estimating probabilities through relative frequency, calculating the theoretical probability from the tree of possibilities, analysis of observed and expected patterns, and construction of simple tables and graphs.
        One of the tasks of the TS RWJ required students to build a tree of possibilities that represented the possible paths of Jefferson's visits to each of five friends located on a 3D board (5 x 5). To enable blind students to build the tree, as well as to perform other tasks, Vita (2012) used a tactile model which she had created and adapted. It will be described in the next section.

This article focuses on analysis of Vita's research results (2012) relating to construction of trees of possibilities by blind students using the tactile model.

METHODOLOGY

In Vita's (2012) three high school blind students (called S2, S3 and S4) was enrolled in a educational experiment abouttheusing of tactile model to learn probabilistic concepts.. The tactile model (Figure 1) was built from five prototypes, and its final version was composed of artifacts and tasks.. The artifacts were: a 3D board (Figura 1), seven plastic shapes with a rectangular base containing 54 square compartments arranged in nine rows and six columns called a hive (Figure 2); 240 cards measuring 2.5 cm x 2.5 cm in textured foam cloth, rubberized and smooth; 300 toys (60 dolls, 60 whistles, 60 yo-yos, 60 rings and 60 belt loops); two covers and a cart.

Figure 1. Final version of the tactile model
Reference: Vita (2012), p. 108.

The experiment was composed by two different king of tasks: tactile recognition and TS RWJ. It is important to highlight students play with tactile model (solving the recognition tasks before the beginning of TS RWJ tasks.(Figure 2).

Figure 2. Tactile recognition of hive
Reference: Vita (2012), p. 174.

The TS RWJ was composed of 23 tasks , divided into three blocks. In the first block called Context , Random testing and Graphing tasks, students should read the following story:

Jefferson and his friends live in the same neighborhood represented by the 3D board. The distance from Jefferson's house to Luana's, Marcos's, Peter's, Orlando's and Aida's is four blocks each. Jefferson used to visit his friends during the week in a pre-established order: Monday, Luana; Tuesday, Marcos; Wednesday, Peter; Thursday, Orlando; and Friday, Aida. To make these visits more exciting, the group agreed that chance would choose the friend visited by Jefferson. To this end, on the way out of his house and at every intersection, Jefferson has to pick one of the two covers; if it was the rubberized one, he will walk one block to the North, and if it is smooth, one block to the East. Each play represents that Jefferson walk one block with an obligatory stop at the crossroads. Jefferson must choose the covers four times to get to the home of one of his friends.

After reading the story, it was asked to students compare the old and new strategies to visit Jefferson' friends, identify the possible results (north ou east) when they threw the covers (instead of coins) and the chance of get heads and tails, carry out a random experimentation, including registering the observed frequencies in the hive by using the toys corresponding to each of the five friends (Figure 3a). It should be noted that the toy used to represent Luana was a doll, Marcos a yoyo, Peter a whistle, Orlando a ring, and Aida a belt loop.

After all, students should be able to construct a 3D pictogram with observed frequencies (Figure 3b)

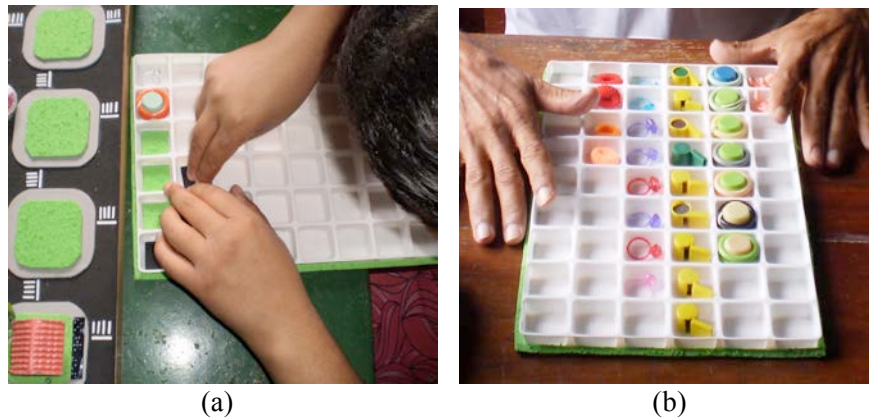| (a)   (b) |

Figure 3. A example of one friend visited (a) e Pictogram 3D (b)
Reference: Vita (2012), p. 193 e 195.

In the second block of tasks, named Mathematics Modeling of possibilities and Graphics representation, the students were asked to organize a tree of possibility that represented all possible paths for Jefferson to visit each of his five friendsFor the construction of the tree the students used the following artifacts: the hives that served as a reference guide; the cards with two different textures (rubberized and smooth), representing Jefferson walking to the East or to the North; and the toys corresponding to each of the five friends represented the expected frequencies. It should be noted that the toy used to represent Luana was a doll, Marcos a yoyo, Peter a whistle, Orlando a ring, and Aida a belt loop(Figure 6).

The students were asked to answer other questions of the second block of tasks such as "Do the five friends have the same chance to be visited by Jefferson?" (by looking at tree diagram of possibilities)

In the third block of tasks, named Compare the different ways of get the probability, the students were asked to compare the relative frequency with the probability. In this paper, it is presented only the results of construction of tree diagram of possibilities.

The meetings for the organization of the tree took place in multifunctional resource classrooms, in which data were collected from the video of the oral application of tasks by Vita (2012), individually applied.

It should be noted that Vita's (2012) use of the tactile mock-up was based on Cazorla and Santana (2010), Cazorla, Gusmão and Kataoka (2011), Hernández, Kataoka and Oliveira (2010) and Ferreira (2011), who worked with students or teachers who could see and used a 2D poster, a coin (head -C- indicating North and tail -X- for East), and a reticule for building the tree of possibility (Figure 4).
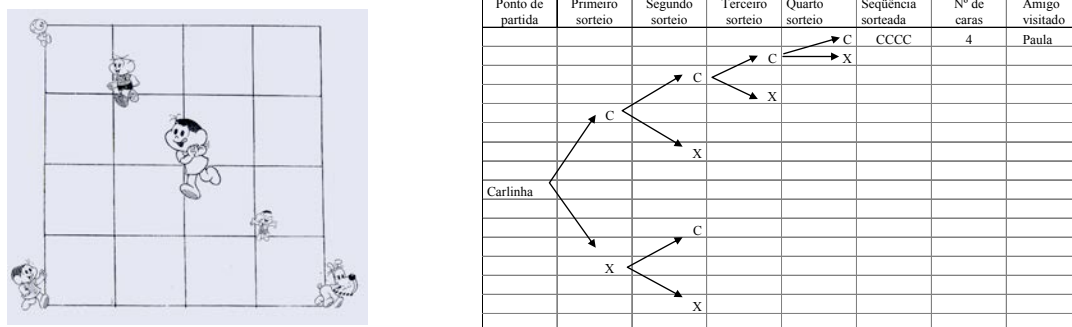

Figure 4. 2D poster and reticule for the tree of possibilities

ANALYSIS AND DISCUSSION OF THE CONSTRUCTION OF THE TREE OF POSSIBILITIES

Analyzing the construction of the tree of possibilities(Figure 6), we found that the students used the 3D board constantly to identify possible paths for Jefferson to visit each of his five friends. When they identified a path, they recorded it in the hive with the cards and placed the toy which corresponded to the visited friend there (Figure 3a).

They could not perform the task simply by swapping the position of the cards to identify all possible combinations and noting a pattern representing the possible paths for visiting every one of the friends, for instance, Jefferson should walking two blocks to the north and two blocks to the east to visit Peter, walking one time to the north and three times to east to visit Orlando (Figure 5).
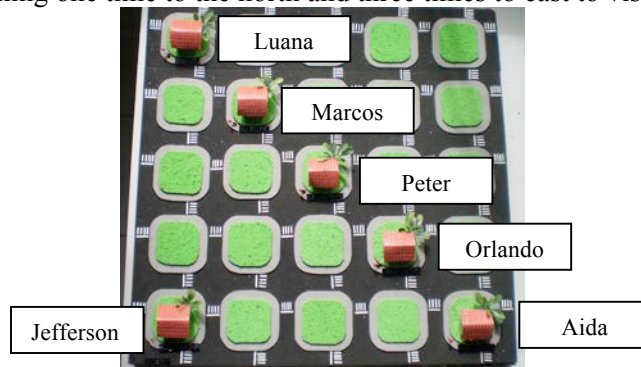

Figure 5. Friends' house at board 3D
Reference: Vita (2012), p. 182.

The greatest difficulty showed by three students was registering the possible paths to visit Peter, the friend living in the Center of the neighborhood. Of the six possible paths, which corresponded to combinations involving two foam smooth cards (black) and two rubberized cards (green), the students were able to identify four paths.

At that point Vita (2012) needed to make an intervention so that all paths were identified and registered, an then the tree diagram of possibilities was constructed , as we can observe in S2's record in Figure 6..
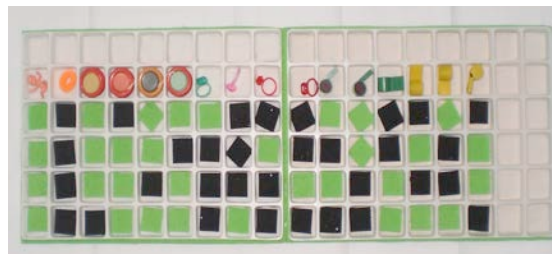

Figure 6. Tree of possibilities of S2
Reference: Vita (2012), p. 199

The difficulties facing these blind students were also observed with not blind students or teachers in the studies of Cazorla, Gusmão and Kataoka (2011), Hernández, Kataoka and Oliveira (2010) and Ferreira (2011). In the research by Cazorla, Gusmão and Kataoka (2011), a couple of math teachers failed to identify all possible paths. Hernández, Kataoka and Oliveira (2010), who worked with 91 students in the third year of high school, reported that 31.7% failed to find a relation on the paths heading to each friend's house. In Ferreira (2011) the teacher-researcher had to make an intervention during the activities of the construction of the tree of possibilities by three pairs of students (referred to as D1, D2, D3) in high school. D1 drew the tree, but had difficulty counting the visited friends. D2 had no difficulty with the composition of the tree nor with making a relationship between the sequences and the friends to be visited, but made a mistake by counting five visits to a friend who had four, and made five visits to the friend in the Center, instead of six. D3 had great difficulty and did not complete the tree.

It is important to reflect that in the case of blind students, unlike students and teachers who can see, they did not have a reticule, and neither one of the paths to serve as an example (see Figure

4). They had to start construction of the tree without any idea what it was because it had not yet been used in school; moreover, any type of graph or different representation was always provided readymade by the teacher.

FINAL CONSIDERATIONS

It was observed a initial difficulty of blind students to develop the tasks of TS RWJ, but the role of the researcher (helping them) was essential to allow constructing the tree diagram of possibilities and then concluding successfully the experiment, , as in Ferreira (2011). The researcher's intervention seemed to us to be justified and essential for promoting students' learning and it is supported by the recommendations of the NCP: CA (Brasil, 1998a), which suggest that SEN students have to be aided, having as a principle the adaptations in teaching method and didactic organization.

These results confirm Vygotsky' idea (1998): he argues that blind child is able to develop her knowledge, regardless of her physical restriction. But, it is necessary to transform the educational environment to be helpful to special necessities of this kind of student.

Finally, we believe that this study could promoting discussion about adaptations of materials and teaching methods of statistical and probabilistic content. These adaptations could allow blind students to learn scholar contents in a suitable way according to their restrictions.

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